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# Flavour Equilibration in Quark-Gluon Plasma

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## Abstract

Within the framework of a dynamical and physically transparent model developed earlier, we study the time evolution of various quark flavours in the baryon-free region in ultrarelativistic heavy ion collisions. We show that even under optimistic conditions, the quark-gluon system fails to achieve chemical equilibrium.

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Quantum Chromodynamics (QCD), the standard theory of strong interactions, predicts that at very high temperature and/or density, the colourless hadronic matter dissolves into its coloured constituents, the quarks and gluons, such that the bulk properties of strongly interacting matter are governed by these degrees of freedom. Such a locally colour-deconfined phase of matter is known as Quark Gluon Plasma (QGP). It is expected that the temperature and density achievable in ultrarelativistic collisions of heavy ions is favourable for QGP formation, although transiently.

Many aspects of this transition, *e.g.*, the order of the phase transition, the value of the critical temperature, signals of the transition, thermodynamic equilibration, are still uncertain. So far, most of the works assume thermodynamic equilibrium throughout the evolution history, after an initial proper time  $\tau_i$  ( $\sim 1$  fm/c). Thus the thermodynamic quantities like pressure, entropy and temperature have direct meaning with respect to an equilibrated system. It has however been recently shown that the approach to equilibrium in ultrarelativistic heavy ion collisions proceeds through a succession of many time scales[1, 2]. In particular, the dominance of the  $gg$  cross section over the  $qq$  or  $q\bar{q}$  cross sections [3] was argued to imply that the gluons equilibrate among themselves appreciably earlier than the whole system of quarks and gluons. In a recent work [1], referred to as **1**, we showed that the time scale of *kinetic* equilibration for the light quarks ( $u, d$ ) is  $\sim 1$  fm/c; for massive quarks, it increases with the mass to rather large values. The significance of these considerations for QGP diagnostics has been discussed in [4, 5]; the emission of particles from the pre-equilibrium era ( $\tau_g \leq \tau \leq \tau_{th}$ ),  $\tau_{th}$  denoting the time for full

thermodynamic equilibrium, may indeed populate the invariant mass or  $p_T$  windows thought to be relevant for signals from the thermalised QGP.

In this letter, we study the *chemical* evolution of the quarks in the system from the epoch  $\tau_g$  ( the proper time when gluons thermalise) onward. Chemical equilibration has very important implications for the signal of QGP formation [4, 5, 6]. It would be ideal to study this problem in the QCD based kinetic theory and attempts along these lines have been made recently [7]. Such calculations, appealing as they are, rely heavily on rather involved numerical simulations and also suffer from dependence on parameters needed to mimic the non-perturbative effects. We propose, as in **1**, a physically transparent scenario which retains the essence of the kinetic theory to the following extent : the gluons thermalise *completely* at a proper time  $\tau_g$  earlier than the quarks. The gluons carry about half of the total momentum and the quarks carry a tiny fraction. We restrict ourselves to the central rapidity region where the number of valence quarks is assumed to be negligibly small.

In order to treat the equilibration of the species one must follow the microscopic evolution of the phase space distribution function  $f(x^\mu, p^\mu)$ , governed by the Boltzmann transport equation :

$$\hat{L}\{f_q\} = \hat{C}\{f_q\} \quad (1)$$

where  $\hat{L}$  is the Liouville operator and  $\hat{C}$  the collision operator, the subscript  $q$  denoting a quark flavour.

The Boltzmann equation is a non-linear, integro-differential equation for the phase space distribution function of the particles. For our present purpose, however, the problem can be addressed in a rather simple manner without losing much information on the time scales for chemical equilibrium.

In ref. [8, 9] the decoupling of the relic particles in the early universe was studied by integrating the Boltzmann equation over the momentum of the particle to obtain an equation for the evolution of number density. We use a similar approach to look at the number density evolution of the partons in ultrarelativistic collisions of heavy ions; the major difference between the two cases is that the expansion dynamics in the early universe (*big bang*) is governed by the Hubble expansion [9] whereas in the case of heavy ion collision (*mini bang*), the expansion dynamics is assumed to be governed by the boost invariant (Bjorken) solution of relativistic hydrodynamics [10]. The equation for the number density evolution then reads

$$\frac{dn_q}{d\tau} + \frac{n_q}{\tau} = \frac{g_q}{(2\pi)^3} \int \{\hat{C}\} \frac{d^3p}{E} \quad (2)$$

where  $n_q$  is non-equilibrium quark density,  $\tau$  the proper time and  $g_q$  the statistical degeneracy of the quarks of flavour  $q$ .

The most important contributions to the collision term in eq. (2) are from gluon fusion ( $gg \leftrightarrow q\bar{q}$ ) and gluon decay ( $g \rightarrow q\bar{q}$ ) [11, 12]. Since we have assumed complete equilibration for the gluons, reactions like  $gg \rightarrow ggg...$  are not included in our work, the gluon density being determined by the temperature of the system at each instant. We ignore the quantum effects but take into account relativistic effects because of the very high temperature. Thus gluons are described by relativistic Maxwell - Boltzmann statistics,  $f_g(E) = g/(2\pi)^3 e^{-E/T}$ . Then eq.(2) can be written as,

$$\frac{dn_q}{d\tau} = -\frac{n_q}{\tau} - R_{gg \rightarrow q\bar{q}}(T) \frac{(n_q^2 - n_{eq}^2)}{n_{eq}^2} + R_{g \rightarrow q\bar{q}}(T) \quad (3)$$

where  $n_{eq}(T) = g_q/(2\pi)^3 \int d^3p/(e^{E/T} + 1)$ , is the equilibrium density. Eq.(3) is a particular form of the Riccati equation; there is no closed form solution of this equation.

In deriving eq.(3) we have assumed T (or CP) invariance, *i.e.*,  $|M|_{gg \rightarrow q\bar{q}} = |M|_{q\bar{q} \rightarrow gg}$ .  $R_{gg \rightarrow q\bar{q}}(T)$  is the quark production rate per unit four volume by the reaction  $gg \rightarrow q\bar{q}$  and  $R_{g \rightarrow q\bar{q}}$  gives the same for the gluon decay,  $g \rightarrow q\bar{q}$  [11, 12]. The thermal quark masses have been taken into account [1] using  $m_q^2(T) = m_q^2 + g^2 T^2/4.5$  in all the rates which was neglected in the work of Biró *et al* [13]. The full derivation of the above equation has been omitted here for the sake of brevity; it will be reported elsewhere [14]. Eq.(3) is a very convenient form of the Boltzmann transport equation with all the relevant features for our present purpose. The first term on the right hand side of the equation represents the "dilution" of the density due to one dimensional expansion, the destruction of the quarks is propotional to the annihilation rates of  $q\bar{q}$  and destruction is balanced by creation through the inverse reaction when  $n_q = n_{eq}$ . The last term stands for the production of quarks due to gluon decay for which there is clearly no inverse process.

To solve eq.(3) for  $n_q(\tau)$  we need to know  $T(\tau)$ , which is obtained by solving the energy conservation equation

$$\frac{d\epsilon}{d\tau} + \frac{\epsilon + P}{\tau} = 0 \quad (4)$$

where  $P$  is the pressure and  $\epsilon$  the energy density. An equation of state relating  $P$  and  $\epsilon$  can be parametrized in the form

$$P = c_s^2 \epsilon \quad (5)$$

Then, for the non-equilibrium configuration [15], we can write

$$\epsilon = [ar_g + br_q(\tau)] T^4 \quad (6)$$

where  $a = (8\pi^2)/45 c_s^{-2}$  and  $b = (7\pi^2)/60 N_f c_s^{-2}$ . In chemical equilibrium,  $r_q(\equiv n_q/n_{eq})$  for all species should be unity and in correspondence with our premise, we take  $r_g=1$  all through.  $N_f$  is the *effective* number of *massless* flavours; appearance of quark masses amounts to reducing the actual number of flavours [16, 17]. For the present, we confine our attention only to the three lightest flavours,  $u, d$  and  $s$  [18].

We then have,

$$\frac{dT}{d\tau} = \frac{bT}{4(an_{eq} + bn_q)} \left[ \frac{n_q}{\tau} + R_{gg \leftrightarrow q\bar{q}}(T) \frac{(n_q^2 - n_{eq}^2)}{n_q^2} - R_{g \rightarrow q\bar{q}}(T) \right] - \frac{T}{3\tau} \quad (7)$$

To solve eq.(3), we need to specify the initial conditions. The initial values of  $n_q(\tau_g)$ 's are obtained from the integration of the structure function [19],

$$n_q = 2A \times \frac{\int_{x_{min}}^1 u_q(x) dx}{\pi R_A^2 \tau_g} \quad (8)$$

where  $A$  is the mass number of the colliding nuclei (208 in our case for Pb),  $x_{min}$  is the minimum value of  $x$ , taken to be 0.02 [1, 2], dictated by the applicability of perturbative QCD. The values of  $n_q(\tau_g) = 1.42, 2.54$  and  $3.05$  for SPS, RHIC and LHC respectively. The corresponding initial values of the temperature ( $T_g$ ) are 330, 500 and 660 MeV at proper times  $\tau_g = 0.54, 0.3$  and  $0.25$  fm/c, respectively [2].

The solutions of eqs.(3) and (7), obtained by the Runge-Kutta method, provide us with the non-equilibrium density  $n_q(\tau)$  and the cooling law,

$$T^3\tau = \left[ \frac{a + br_q(\tau_g)}{a + br_q(\tau)} \right]^{3/4} T_g^3\tau_g \quad (9)$$

As is evident from the above discussions, the non-equilibrium density  $n_q$  has an explicit dependence on  $\tau$  and an implicit dependence on  $\tau$  through  $T(\tau)$ . But the equilibrium density  $n_{eq}(T)$  has only an implicit dependence on  $\tau$  through  $T(\tau)$ . The ratio  $r_q$  thus assumes an *universal* feature, since the implicit time dependence gets eliminated. The time dependence of the ratio  $r_q$  can then be used as a ready marker for chemical equilibrium; the time at which the explicit time dependence of  $r_q$  vanishes corresponds to the time for chemical equilibration for the flavour  $q$ .

In contrast to the earlier work of Biró *et al* [13], we have not included reactions like  $q\bar{q} \rightarrow Q\bar{Q}$ , since the initial system is dominated by gluons[2, 7]; the quark density is very low compared to the gluons. These reactions are suppressed by a factor of about 1/16 [5] compared to the case when the quarks are in complete equilibrium. Biró *et al* [13] also estimated their initial density from the HIJING model [20].

The ratios  $r_q = n_q/n_{eq}$  are plotted in fig.1 for SPS, RHIC and LHC energies. At RHIC and LHC energies, the ratios for  $u(d)$  quarks and  $s$  quarks are the same at early times but at late times the  $u(d)$  quarks dominate over the  $s$  quarks. The reason is that at early times the thermal mass ( $\propto T$ ) dominates over the current quark mass but at later times the thermal mass becomes small and the main contribution comes from the current quark mass which is large for  $s$  quarks ( $m_s \sim 150 MeV$ ) compared to the  $u(d)$  quark masses ( $m_{u(d)} \sim 10 MeV$ ); there occurs thus some Boltzmann suppression for  $s$  quarks relative to  $u(d)$  quarks. At SPS energies, at early times the ratio for  $s$  quarks is large compared to the  $u(d)$  quarks, this is due to the initial normalisation. The non-equilibrium density  $n_{u(d)} \sim n_s$  but the equilibrium density for  $s$  quarks is small compared to  $u$  and  $d$  quarks due to larger mass.

At small time scales (larger temperature) the slope of the ratio for LHC energies is opposite to that of SPS and RHIC. The non-equilibrium density contains the effect of expansion as well as production, whereas the equilibrium density is affected by expansion only. In the ratio, the effect of production dominates over the expansion at LHC i.e. larger temperature, so the ratio increases. However at RHIC and especially at SPS the quark production is not so intense to dominate over expansion.

In fig.1, we have also shown the time variation of temperature, given by the cooling law (eq.(9)). The lifetime of the quark-gluon system is the time when  $T$  falls below  $T_c$ , taken to be 160 MeV as an optimistic estimate. It is abundantly clear that under either of the two criteria,  $r_q \rightarrow 1$  and/or flatness of  $r_q$  - the quarks remain out of chemical equilibrium in the baryon-free region at SPS, RHIC and LHC energies. Detailed account of these effects in QGP diagnostics must be considered; work along this line is in progress.

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## Figure Captions

Figure 1: Ratio of non-equilibrium density to equilibrium density and temperature as functions of time at SPS, RHIC and LHC energies.